Turn Performance

by David F. Rogers

dfr@usna.navy.mil
http://www.nar-associates.com
http://web.usna.navy.mil/~dfr/rogers.html

With this article we start a new topic on turning flight. Turning flight comes up in the discussion of the chandelle, which is a required part of the commercial certificate, in mountain flying when you fly up that proverbial box canyon and need to turn around, and in that wonderful circle to land maneuver in instrument flying, to name only a few.

Turning flight is somewhat more complex than steady straight and level flight, because there are radial accelerations and the aircraft is banked. Figure 1 is a diagram showing the forces acting on the aircraft in turning flight. A bit complicated, isn't it? Well don't despair (or quit reading), we're going to work on it. See the rectangle that looks tipped a bit because of the bank angle, ϕ (phi). The lift, L, drag, D, and thrust, T, vectors lie in the plane of that rectangle if we assume a coordinated turn, i.e., if we keep the ball in the middle. With a Bonanza that is not too hard, because the rudder–aileron interlock puts most of the required rudder in for you when you deflect the ailerons.

The weight, W, still acts vertically. Thus, the total lift is *not* available to counteract the weight, because the lift is tipped off to one side because of the bank angle. We find the part (component) of the lift that counteracts the weight by breaking the lift up into pieces. The piece that is vertical is $L\cos\phi$. It is this piece that opposes the weight. The other part of the lift acts toward the center of the turn, i.e., radially, and is $L\sin\phi$. This part of the lift opposes the centrifugal force called C.F. in the diagram. You remember that is the force that keeps the water in the pail when you swing it around in a vertical circle! Yeah sure. It's like every time I try to walk on water I keep getting my socks wet!

There are two other force components that we must take into account. These force components are caused by the fact that the thrust is inclined to the flight path by an angle, α_T (alphaT). They are shown in Figure 1. Note that α_T is not necessarily the angle of attack. One thrust component is in the vertical direction, and the other is in the radial direction. The thrust component in the vertical direction is $T\sin\alpha_T\cos\phi$, while the thrust component in the radial direction is $T\sin\alpha_T\sin\phi$.

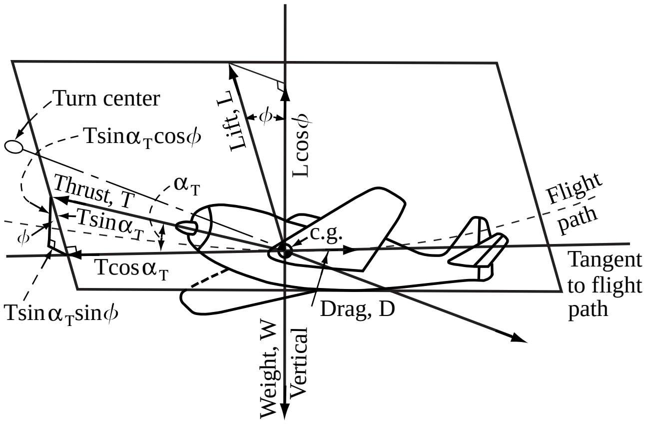

Figure 1. Forces in a steady level coordinated turn.

And now comes the math. You knew I was going to do that, didn't you? Well, we need to do the math so that everyone knows where we are coming from. Looking at Figure 1, we want to sum all the forces in the vertical direction. We have two positive forces, the component of lift, $L\cos\phi$ and the thrust component, $T\sin\alpha_T\cos\phi$. There is one negative component, the weight, W. Well, if this is going to be a steady turn, then the sum of the positive forces must equal the negative force. Thus, our first equation is

$$(L + T\sin\alpha_T)\cos\phi = W \tag{1}$$

Now let's look at the radial direction. Here again there are two positive forces, the component of lift in the radial direction, $L\sin\phi$, and the thrust component, $T\sin\alpha_T\sin\phi$. The single negative component is the centrifugal force, C.F. Again letting the sum of the positive forces equal the negative force we have

$$(L + T\sin\alpha_T)\sin\phi = C.F. \tag{2}$$

That wasn't too hard was it?

Now the hard bit, centrifugal force. Centrifugal force, C.F., is the mass of the aircraft, which is simply the weight divided by the acceleration of gravity, W/g, times the radial acceleration. The radial acceleration is the velocity (true airspeed, TAS) squared divided by the radius of the flight path, V^2/R. So the second equation, Eq.(2) becomes

$$(L + T\sin\alpha_T)\sin\phi = \frac{W}{g}\frac{V^2}{R} \tag{3}$$

Don't go away, we're almost there. Look at the expression in the brackets, ( ), on the left hand sides of Eqs.(1) and (3). Notice that they are the same. Also notice that the weight, W, appears on the right hand side of both equations. If we divide one equation by the other, then the expression in brackets, ( ), and the weight disappear. The result is

$$\tan\phi = \frac{V^2}{gR} \tag{4}$$

or rearranging the equation the turn radius is

$$R = \frac{V^2}{g \tan \phi} \tag{5}$$

This is a very important result. Equation(5) says that the radius of a steady level turn depends only on the velocity in the turn and the bank angle *for any aircraft*, be it a Piper Cub, a Bonanza, or a Boeing 747! Now you can forget the math, and just remember this result.

Figure 2 shows the turn radius for various bank angles from 10° to 60° plotted against true airspeed. For a bank angle of zero degrees the turn radius is infinite, i.e., you are in straight and level flight. Conversely, when the bank angle is ninety degees your turn radius is zero! This is why, in addition to the 60° limitation on nonaerobatic flight, the graph only goes to a maximum bank angle of 60°. Notice from Figure 2 that for a given TAS the turn radius *decreases* with *increasing* bank angle. Furthermore, for a given bank angle the turn radius *decreases* with *decreasing* true airspeed. From Figure 2 we conclude that to achieve the minimum turn radius we want to fly as close as comfortable to stall velocity, and at as high a bank angle as is comfortable. As an example, for a TAS 10% above the stall velocity and a bank angle of 45° the radius in a steady level turn is

$$R = 0.0813V^2 \text{ (ft) with } V \text{ in mph} \tag{6}$$

or

$$R = 0.107V^2 \text{ (ft) with } V \text{ in kts} \tag{7}$$

For a stall velocity (TAS) of 73 mph the turn radius is 433 ft! The acceleration of gravity, g, is 32.174 ft/sec^2. Also don't forget to convert the velocity to ft/sec by multiplying miles per hour by 1.47 and knots by 1.69. For those of you that don't have access to a scientific calculator, Table 2 gives the values of $1/\tan\phi$.

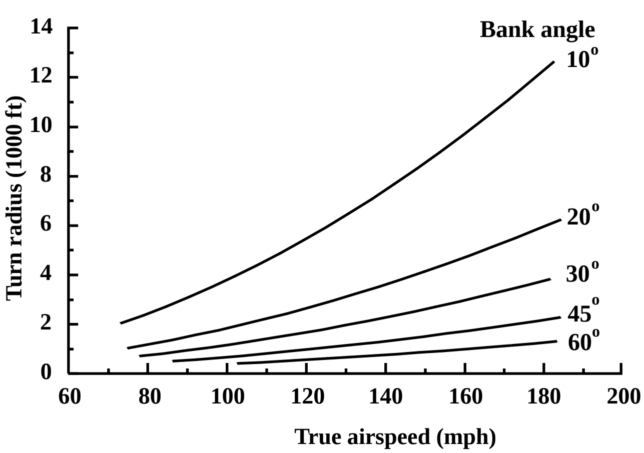

Figure 2. Radius of a steady level turn.

Table 2. $1/\tan\phi$ for Various Values of ϕ

ϕ	$1/\tan\phi$	ϕ	$1/\tan\phi$
0	Infinite40	1.19	
5	11.43	45	1.00
10	5.67	50	0.84
15	3.73	55	0.70
20	2.76	60	0.58
25	2.14	65	0.47
30	1.73	70	0.36
35	1.43	75	0.27

Although Eq.(5) says that in a steady level turn the radius depends only on the velocity (TAS), you do have to maintain that assumed steady level turn! We all know that the stall velocity increases as the bank angle increases. Hence, the stall velocity in the turn provides an obvious limit. In addition, the power required to maintain the steady level turn also increases as the bank angle increases; hence the power available also limits turn performance. We'll look at these effects, as well as others, in future articles. We will also investigate the effects of removing the level coordinated turn restriction.